2020
MATHEMATICS
[HONOURS]
Paper : VI

Full Marks : 100 Time : 4 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and Symbols have their usual meanings.

1. Answer any **ten** questions from the following: $2\times10=20$

i) Define angular momentum of a system of n particles about a point O.

ii) State D'Alembert's principle.

iii) Show that the distance between any two points is invariant under Galilean Transformations.

iv) State the theorem of perpendicular axes.

v) Write down the stress matrix at a point in a fluid in equilibrium.

vi) State the energy test of stability.

vii) What is the form of inertia matrix with respect to principal axes?

viii) Write down the dimensions of shearing stress and torque.

ix) What is equi-pressure surface? Write down its equation.

x) State the principle of virtual work under coplanar forces.

xi) Distinguish between ideal fluid and viscous fluid.

xii) What are the conditions of equilibrium for freely floating bodies?

xiii) Write down the work done by a force $\vec{F}$ in a moving particle round a closed curve C. What happens, when the force field is conservative?

xiv) What is Poinsot's Central axis? Write down its Cartesian equation.

xv) Define centre of pressure of a plane lamina.

xvi) What do you mean by specific heat at constant volume and specific heat at constant pressure?

2. Answer any **five** questions: $8\times5=40$

a) i) Show that the angular momentum of a system of n particles about a fixed point is equal to the angular momentum of the total mass concentrated at the centre of mass plus the angular momentum of the system about its centre of mass.

ii) Obtain the inertia matrix for a homogeneous rectangular lamina of mass M bounded by $x=\pm a, y=\pm b$ with respect to the coordinate axes. 4+4

b) i) The lengths AB and AD of the sides of a rectangle ABCD are 2a and 2b. Show that the inclination to AB of one of the principal axes at A is

$$\frac{1}{2}\tan^{-1}\frac{3ab}{2\left(a^2-b^2\right)}.$$

ii) Show that the kinetic energy of a rigid body moving in two dimensions is $\frac{1}{2}Mv^2+\frac{1}{2}MK^2\dot{\theta}^2$, the notations involved are to be explained by you. 4+4

c) A uniform rod is held at an inclination α to the horizon with one end in contact with a horizontal table whose coefficient of friction is μ. If it be then released, show that it will commence to slide if

$$\mu<\frac{3\tan\alpha}{1+4\tan^2\alpha}.$$ 8

d) A uniform solid cylinder is held at rest with its axis horizontal on a plane whose inclination to the horizon is α. If it be then released to roll down a length L of the slope, show that the velocity of its centre of mass is then

$$\sqrt{\frac{4}{3}gL\sin\alpha}.$$ 8

e) A sphere is projected with an underhand twist down a rough inclined plane. Show that it will turn back in the course of its motion, if

$$2a\omega\left(\mu-\tan\alpha\right)>5u\mu,$$

where u, ω are the initial linear and angular velocities of the sphere, μ being the coefficient of friction and α the inclination of the plane to the horizon. 8

f) Find the condition that a given straight line may be a principal axis at any point on the line, and if so then determine the other two principal axes. 8

g) Explain the concept of momental ellipsoid and then find the equation of the momental ellipsoid at the centre of an elliptic plate. 4+4

3. Answer any **two** questions: 8×2=16

a) A heavy uniform flexible string is hanging between two fixed points in a vertical plane. Obtain the cartesian equation of the curve in which the string hangs. Explain the significance of the parameter of the curve. 6+2

b) A force parallel to the axis of z acts at the point $(a, 0, 0)$ and an equal force perpendicular with the axis of z acts at the point $(-a, 0, 0)$. Show that the central axis lies on the surface

$$z^2\left(x^2+y^2\right)=\left(x^2+y^2-ax\right)^2. \qquad 8$$

c) Two uniform rods AB, BC of weights W and W' are smoothly jointed at B and their middle points are joined across by a cord. The rods are tightly held in a vertical plane with their ends A, C resting on a smooth horizontal plane. Show by the principle of virtual work that the tension of the cord is

$$\frac{(W+W')\cos A\cos C}{\sin B}. \qquad 8$$

4. Answer any **three** questions: 8×3=24

a) Show that a mass of fluid is at rest under the forces

$$X=(y+z)^2-x^2$$
$$Y=(z+x)^2-y^2$$
$$Z=(x+y)^2-z^2$$

and $x^2+y^2+z^2-2yz-2zx-2xy=\text{constant}$ are the equations of the equi-pressure surfaces. 4+4

b) A triangular lamina ABC is immersed with side AB in the surface of a liquid and side AC vertical in the liquid whose density varies as the depth. If AB=a and AC=b, show that the position of the centre of pressure is $\left(\frac{a}{5}, \frac{3b}{5}\right)$ referred to AB and AC as the coordinate axes. 8

c) Obtain the necessary and sufficient condition of equilibrium of a fluid of the form

$$\vec{F}.\left(\vec{\nabla}\times\vec{F}\right)=0,$$

the symbols are to be explained by you. 8

d) ABC is a triangular lumina immersed vertically in water with C in the surface and the sides AC, BC are equally inclined to the surface. Prove that the vertical through C divides the triangle into two others, the thrusts on which are in the ratio $\left(b^3+3ab^2\right):\left(a^3+3a^2b\right)$. 8

e) i) A gas satisfying Boyle's law $p=k\rho$ is acted on by forces

$$X=-\frac{y}{x^2+y^2},\ Y=-\frac{x}{x^2+y^2}$$

Show that density varies as $e^{\frac{\theta}{k}}$, where

$\tan\theta=\frac{y}{x}$.

ii) A gas is equilibrum under the laws $p=k\rho^\gamma=R\rho T$, where p, ρ, T are the pressure, density and absolute temperature respectively, and k, γ, R are constants. Prove that temperature decreases upward at a constant rate.

$$\frac{\gamma-1}{\gamma}.\frac{g}{R}.$$

4+4

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